

*Greenhouse gas emission uncertainty  
in compliance proving and emission trading*

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# ***Overview***

- Proving compliance under uncertainty
  - 1. Problem formulation
  - 2. Undershooting
  - 3. Adjustment of the target level
- Emission trading under uncertainty
  - 1. Effective traded emission
  - 2. Effective emission permits

## ***Compliance proving***

real (unknown) emission      estimated emission

basic year

$$x(t_b)$$

$$\hat{x}(t_b)$$

commitment year

$$x(t_c)$$

$$\hat{x}(t_c)$$

$$x(t_c) - (1 - \delta)x(t_b) \leq 0 \quad \hat{x}(t_c) - (1 - \delta)\hat{x}(t_b) \quad ?$$

$\delta$  – fraction of emission to be reduced

## ***Interval uncertainty***

$\pm\Delta$  - uncertainty interval

$$\hat{x}(t_b) - \Delta_b \leq x(t_b) \leq \hat{x}(t_b) + \Delta_b$$

$$\hat{x}(t_c) - \Delta_c \leq x(t_c) \leq \hat{x}(t_c) + \Delta_c$$

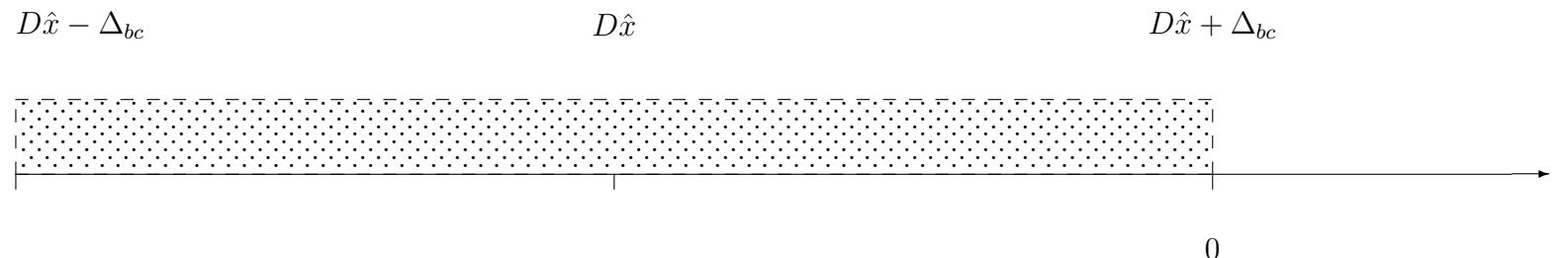
$\Downarrow$

$$D\hat{x} - \Delta_{bc} \leq x(t_c) - (1 - \delta)x(t_b) \leq D\hat{x} + \Delta_{bc}$$

$$D\hat{x} = \hat{x}(t_c) - (1 - \delta)\hat{x}(t_b)$$

$$\Delta_{bc} = \Delta_c + (1 - \delta)\Delta_b$$

## ***Undershooting (full credibility)***

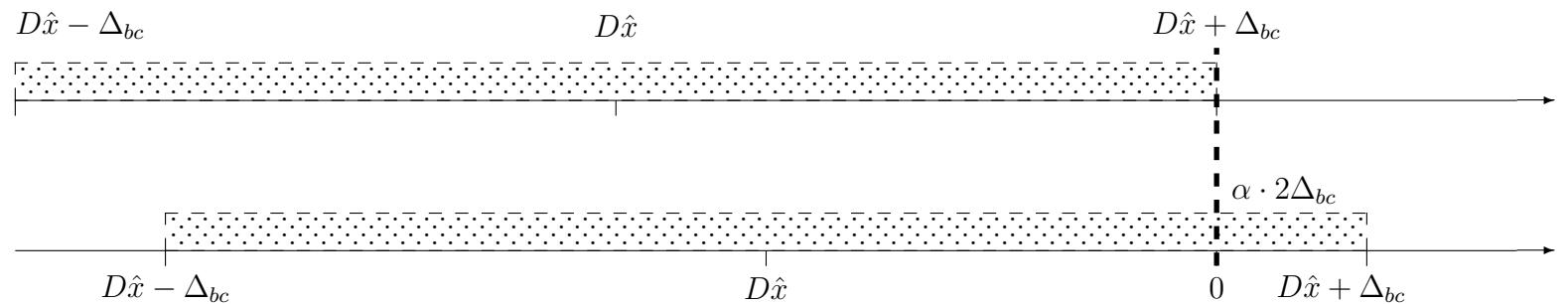


$$D\hat{x} + \Delta_{bc} = \hat{x}(t_c) - (1 - \delta)\hat{x}(t_b) + \Delta_{bc} \leq 0$$

or

$$\hat{x}(t_c) + \Delta_{bc} \leq (1 - \delta)\hat{x}(t_b)$$

## **Compliance with risk $\alpha$ (partial credibility)**



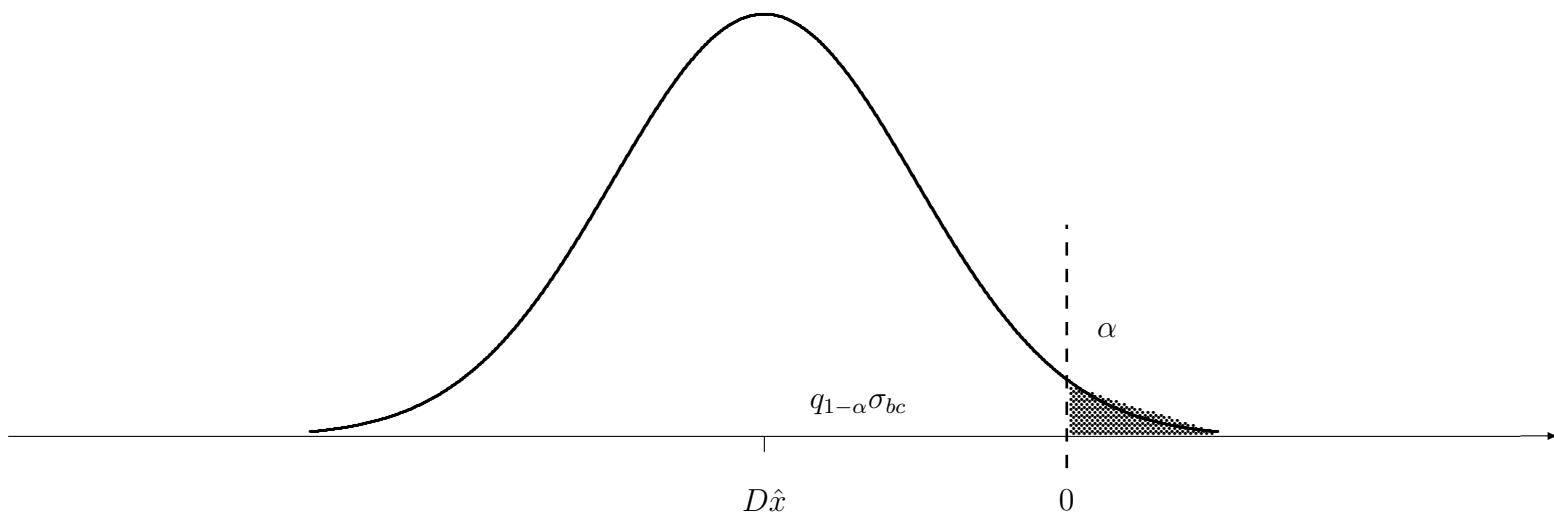
$$\hat{x}(t_c) + (1 - 2\alpha)\Delta_{bc} \leq (1 - \delta)\hat{x}(t_b)$$

$\alpha$  – assumed risk that the party is non-compliant

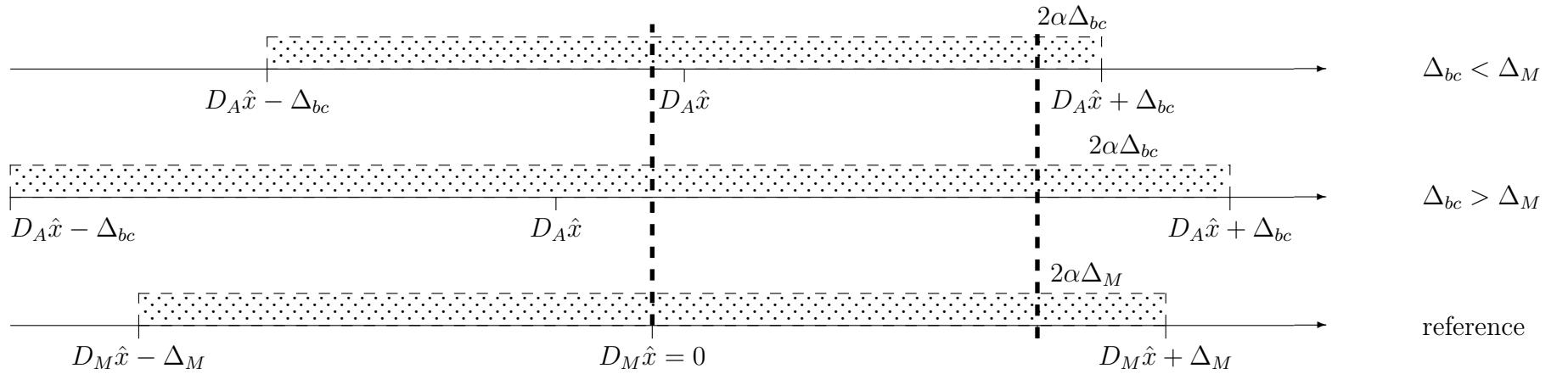
$$\hat{x}(t_c) \leq \left(1 - \delta - (1 - 2\alpha)R_{bc}\right)\hat{x}(t_b)$$

$R_{bc} = \frac{\Delta_{bc}}{\hat{x}(t_b)}$  – relative uncertainty

## *Stochastic type uncertainty*



## Adjusted target level



$$\hat{x}(t_c) \leq \left(1 - \delta - (1 - 2\alpha)(R_{bc} - R_M)\right) \hat{x}(t_b)$$

$$R_{bc} = \frac{\Delta_{bc}}{\hat{x}(t_b)} \quad R_M = \frac{\Delta_M}{(1-\delta)\hat{x}(t_b)} - \text{relative uncertainties}$$

## **Choice of $R_M$**

mean relative  
uncertainty

mean absolute  
uncertainty

$$R_M = \frac{1}{N} \sum_{i=1}^N R_{bc}^{(i)}$$

$$R_M = \frac{\sum_{i=1}^N \hat{x}^{(i)}(t_b) R_{bc}^{(i)}}{\sum_{i=1}^N \hat{x}^{(i)}(t_b)}$$

## ***Emission trading – the selling party***

- $S$  – the selling party
- $\hat{x}^S(t_c)$  – emission at the commitment year
- $R_c^S = \frac{\Delta_c^S}{\hat{x}^S(t_c)}$  – relative uncertainty
- $\hat{E}^S$  – the unit of emission, e.g MtonCO<sub>2</sub>
- $\hat{E}^S R_c^S$  – uncertainty of the unit of emission
- $n\hat{E}^S$  –  $n$  units of emission sold to other party
- $n\hat{E}^S R_c^S$  – uncertainty of  $n$  units of emission sold to other party

## ***Emission trading – the buying party***

- $B$  – the buying party
- $\hat{x}^B(t_c)$  – emission at the commitment year
- $\Delta_{bc}^B = \Delta_c^B + (1 - \delta^B)\Delta_b^B$  – uncertainty of compliance condition
- $\hat{x}^B(t_c) - n\hat{E}^S$  – emission balance after trade
- $\Delta_{bc}^B + n\hat{E}^S R_c^S$  – uncertainty of compliance condition after trade

## ***Effective traded emission***

Condition before the trade

$$\hat{x}^B(t_c) + (1 - 2\alpha)\Delta_{bc}^B \leq (1 - \delta^B)\hat{x}^B(t_b)$$

Condition after the trade

$$\hat{x}^B(t_c) - n\hat{E}^S + (1 - 2\alpha)[\Delta_{bc}^B + n\hat{E}^S R_c^S] \leq (1 - \delta^B)\hat{x}^B(t_b)$$

difference  $\implies$  effective traded emission

$$nE_{eff} = n\hat{E}^S - n(1 - 2\alpha)\hat{E}^S R_c^S = n\hat{E}^S[1 - (1 - 2\alpha)R_c^S]$$

Conversion of one unit

$$E_{eff} = \hat{E}^S[1 - (1 - 2\alpha)R_c^S]$$

## ***Effective tradable permits***

$\hat{E}$  – the emission permit

$E_{eff}$  – the tradable permit

$\hat{x}(t_c)$  – the allotted emission

$\hat{x}(t_c)[1 - (1 - 2\alpha)R_c] = l(t_c)$   
– the tradable permits

Compliance proving condition – undershooting with risk  $\alpha$

$$l(t_c) \leq [1 - \delta - 2(1 - 2\alpha)R_c]l(t_b)$$

Compliance proving condition – adjustment of the target level

$$l(t_c) \leq [1 - \delta - (1 - 2\alpha)(2R_c - R_M)]l(t_b)$$

## ***Compliance proving and trading scheme***

**Before:** Decision on choice of parameters, like  $\delta, \alpha, R_M$

1. emission in the (consecutive) base year:  $\hat{x}(t_b)$
2. tradable permits in the base year:  $l(t_b) = \hat{x}(t_b)[1 - (1 - 2\alpha)R_b]$
3. allotted tradable permits in the commitment year:

$$l(t_c) = [1 - \delta - (1 - 2\alpha)(2R_c - R_M)]l(t_b)$$

4. allotted emission in the commitment year:  $\hat{x}(t_c) = \frac{l(t_c)}{1 - (1 - 2\alpha)R_c}$

**Permits  $l(t_c)$  traded on normal basis**

## ***Conclusions (1)***

1. The approach solves the “credibility problem” by reducing different quality inventories to common level.
2. The approach reduces the uncertainty in observation problem to the well known “classic” permit trading with simple recalculation of the (estimated) emissions to effective permits and vice versa.
3. The approach is flexible – it has few parameters, which can be appropriately tuned.

## ***Conclusions (2)***

4. The approach encourages further investigations on documenting and decreasing uncertainty in inventories.
5. The approach can be readily adapted to trade between different activity plants with scattered uncertainties.
6. The approach requires additional agreements between parties, which may partially change those done so far.